Workshop: **Algebra 2**

Topics Covered:

- Solving Linear Equations
- Evaluation and Transposition of formulae

Solving Linear Equations

When we talk about linear equations we are referring to equations that are in terms of one variable.

> Definition:
>
> A linear equation is an equation that can be written in the form
>
> $$ax + b = 0$$
>
> where a and b are real numbers.
>
> Linear equations have one root.

To find the solution of the linear equation, get the variable involved on one side and all constants on the other side of the equals sign using inverse operations. The solution can be checked by substituting the value back into the original equation and making sure that the right hand side equals the left hand side.

Example 1: Find the value of x in: $x - 10 = -2$

Solution: $x = -2 + 10$

$x = 8$ →

Check:
LHS: $x - 10$
$= 8 - 10$
$= -2 = \text{RHS}$
$\therefore$ the solution $x = 8$ is correct

Example 2: Find the value of x in the following equation:

$$5x + 2 = 7$$

Solution:

$$5x + 2 = 7$$
$$5x = 7 - 2$$
$$5x = 5$$
$$x = \frac{5}{5}$$
$$x = 1$$

Check:
LHS: $5x + 2$
$= 5(1) + 2$
$= 7 = \text{RHS}$
$\therefore$ the solution $x = 1$ is correct

Example 3: Find the value of x in the following equation:

$$\frac{2}{3}x + 10 = \frac{x}{5} + \frac{36}{5}$$
$$\frac{2}{3}x - \frac{x}{5} = \frac{36}{5} - 10$$
$$x\left(\frac{2}{3} - \frac{1}{5}\right) = \frac{36}{5} - 10$$
$$x\left(\frac{10-3}{15}\right) = \frac{36-50}{5}$$
$$x\left(\frac{7}{15}\right) = \frac{-14}{5}$$
$$\frac{7}{15}x = \frac{-14}{5}$$
$$7x = \frac{-14}{5} \times 15$$
$$7x = -42$$
$$x = -6$$

Check:
LHS: $\frac{2}{3}x + 10$
$= \left(\frac{2}{3} \times -6\right) + 10$
$= \left(\frac{-12}{3}\right) + 10$
$= -4 + 10$
$= 6$

RHS: $\frac{x}{5} + \frac{36}{5}$
$= \frac{-6}{5} + \frac{36}{5}$
$= \frac{-6+36}{5}$
$= \frac{30}{5}$
$= 6$

LHS = RHS and so $x = -6$

is the correct solution

Example 4:

Find the solution to the equation $2(x-1)+x=5(2x+3)-2(x+3)$

Solution:

$$2(x-1)+x=5(2x+3)-2(x+3)$$

$$2x-2+x=10x+15-2x-6$$

$$3x-2=8x+9$$

$$-2-9=8x-3x$$

$$-11=5x$$

$$x=-\frac{11}{5}$$

Check:

LHS: $2(x-1)+x$

$$=2\left(-\frac{11}{5}-1\right)+\left(-\frac{11}{5}\right)$$

$$=2\left(-\frac{16}{5}\right)-\frac{11}{5}$$

$$=-\frac{32}{5}-\frac{11}{5}$$

$$=-\frac{43}{5}$$

RHS: $5(2x+3)-2(x+3)$

$$=5\left[2\left(-\frac{11}{5}\right)+3\right]-2\left(-\frac{11}{5}+3\right)$$

$$=5\left(-\frac{22}{5}+3\right)-2\left(\frac{4}{5}\right)$$

$$=5\left(-\frac{7}{5}\right)-\frac{8}{5}$$

$$=-7-\frac{8}{5}$$

$$=-\frac{43}{5}$$

Therefore LHS = RHS and so $x=-\frac{11}{5}$ is the correct solution.

Questions (Solving linear equations)**:**

Solve the following linear equations:

1. $2 + 3x = 23$
2. $-17 = 16x + 3$
3. $9x + 5 = 7x + 3x - 2$
4. $5(x - 3) + 4x - 1 = 2x + 3(x - 2)$
5. $\frac{1}{2}(x+7) = \frac{3x}{5} + 9$

(Solutions on page 9)

Evaluation and Transposition of formulae

A formula, in mathematics, gives a relationship between different quantities. When there is more then one, we use the word formulae.

When working with formulae, it may be necessary to single out one of the quantities involved in terms of all the others. This procedure is often referred to as *transpose the formula* and make that quantity *the subject of the equation*.

The procedures used to transpose a formula to make a certain quantity the subject is the same as those used to re-arrange linear equations.

Example 1: Make x the subject of the formula $y = 3(x + 7)$

Solution: $y = 3(x + 7)$

Dividing both sides by 3: $\frac{y}{3} = x + 7$

Subtracting 7 from both sides: $\frac{y}{3} - 7 = x$

Therefore $x = \frac{y}{3} - 7$

Example 2: Make g the subject of the formula $t = 2\pi\sqrt{\frac{1}{g}}$

Solution:

$$t = 2\pi\sqrt{\frac{1}{g}}$$

Squaring both sides $\quad t^2 = 4\pi^2 \frac{1}{g}$

Multiplying both sides by g $\quad gt^2 = 4\pi^2$

Dividing both sides by t^2 $\quad g = \frac{4\pi^2}{t^2}$

Therefore $g = \frac{4\pi^2}{t^2}$

Example 3: Make y the subject of the formula $a = b + c\sqrt{x^2 - y^2}$

Solution:

$$a = b + c\sqrt{x^2 - y^2}$$

Subtracting b from both sides $\quad a - b = c\sqrt{x^2 - y^2}$

Dividing by c $\quad \frac{a-b}{c} = \sqrt{x^2 - y^2}$

Squaring both sides $\quad \left(\frac{a-b}{c}\right)^2 = x^2 - y^2$

Subtracting x^2 $\quad \left(\frac{a-b}{c}\right)^2 - x^2 = -y^2$

Multiplying through by -1 $\quad x^2 - \left(\frac{a-b}{c}\right)^2 = y^2$

Taking the square root of both sides: $\quad \pm\sqrt{x^2 - \left(\frac{a-b}{c}\right)^2} = y$

Therefore $y = \pm\sqrt{x^2 - \left(\frac{a-b}{c}\right)^2}$

Example 4: Make x the subject of the formula $y(2x + 1) = x + 1$

Solution: $y(2x + 1) = x + 1$

x appears on both sides so we firstly need to rearrange the formula to get all x terms on one side.

$$y(2x + 1) = x + 1$$
$$2xy + y = x + 1$$
$$2xy - x = 1 - y$$

Factorising the LHS $\quad x(2y - 1) = 1 - y$

Divide both sides by $2y - 1$ $\quad x = \dfrac{1-y}{2y-1}$

Therefore $x = \dfrac{1-y}{2y-1}$

Example 5: Rearrange the formula $\dfrac{y}{y+x} + 5 = x$ to make y the subject.

Solution: $\dfrac{y}{y+x} + 5 = x$

Multiply both sides by $y + x$ $\quad y + 5(y + x) = x(y + x)$

Rearrange to get all y terms on one side

$$y + 5y + 5x = xy + x^2$$
$$6y - xy = x^2 - 5x$$

Factorise the LHS $\quad y(6 - x) = x^2 - 5x$

Divide both sides by $6 - x$ $\quad y = \dfrac{x^2 - 5x}{6 - x}$

Therefore $y = \dfrac{x^2 - 5x}{6 - x}$

Questions (Transposition of formulae)**:**

In each of the following, transpose the given formula to make the symbol in brackets the subject of the formula.

1. $y = 2(w + h)$ (h);
2. $m = k\sqrt{a(1-x)}$ (x);
3. $y = a + \dfrac{1}{1-x}$ (x);
4. $a(3b - 1) = 2b + 2$ (b);
5. $a = \dfrac{2-7b}{3+5b}$ (b);
6. $n = \dfrac{1}{2L}\sqrt{\dfrac{r}{p}}$ (r);

(Solutions on page 10)

In some cases you may be given particular values to evaluate a formula. For example, the area of a circle can be calculated by using the formula:

$$A = \pi r^2$$

where A is the area and r is the radius.

If you are given that the radius of a circle is 4cm, you can calculate the area by substituting r = 4 into the above formula, i.e.

$$A = \pi r^2 = \pi(4)^2 = 16\pi = 50.27\text{cm}^2$$

Example: If v = u + at, find t when v = 35, u = 5 and a = 3.

Solution: $v = u + at$

Substitute in the given values: $35 = 5 + 3t$

Rearrange for t: $35 - 5 = 3t$

$30 = 3t$

$t = 10$

Questions (Evaluation of formulae)**:**

1. If $s = \frac{1}{2}(u + v)t$, find s if $u = 2.6$, $v = 3.2$ and $t = 2.5$
2. If $V = \pi r^2 h$, find r when $V = 275$ and $h = 4$.
3. If $t = a + (n - 1)d$, find n when $a = 5.6$, $d = 5$ and $t = 25.6$.
4. If $f = \frac{vu}{v + u}$, find v when $f = 20$ and $u = 25$.
5. If $F = \frac{m(v - u)}{t}$, find u when $F = 8$, $m = 20$, $v = 4$ and $t = 5$.

(Solutions on page 11)

Solutions (Solving linear equations)**:**

1.

$$2 + 3x = 23$$
$$3x = 21$$
$$x = 7$$

2.

$$-17 = 16x + 3$$
$$-20 = 16x$$
$$x = -\frac{20}{16}$$
$$x = -\frac{5}{4}$$

3.

$$9x + 5 = 7x + 3x - 2$$
$$9x + 5 = 10x - 2$$
$$5 + 2 = 10x - 9x$$
$$7 = x$$

4.

$$5(x - 3) + 4x - 1 = 2x + 3(x - 2)$$
$$5x - 15 + 4x - 1 = 2x + 3x - 6$$
$$9x - 16 = 5x - 6$$
$$9x - 5x = -6 + 16$$
$$4x = 10$$
$$x = \frac{10}{4}$$
$$x = \frac{5}{2}$$

5.

$$\frac{1}{2}(x+7) = \frac{3x}{5} + 9$$
$$(x+7) = \frac{6x}{5} + 18$$
$$5(x+7) = 6x + 18$$
$$5x + 35 = 6x + 90$$
$$5x - 6x = 90 - 35$$
$$-x = 55$$
$$x = 55$$

Solutions (Transposition of formulae)**:**

1.

$$y = 2(w + h)$$
$$y = 2w + 2h$$
$$y - 2w = 2h$$
$$h = \frac{y - 2w}{2}$$

2.

$$m = k\sqrt{a(1 - x)}$$
$$m^2 = k^2 a(1 - x)$$
$$\frac{m^2}{k^2 a} = 1 - x$$
$$\frac{m^2}{k^2 a} - 1 = -x$$
$$x = 1 - \frac{m^2}{k^2 a}$$

3.

$$y = a + \frac{1}{1 - x}$$
$$y(1 - x) = a(1 - x) + 1$$
$$y - xy = a - ax + 1$$
$$ax - xy = a + 1 - y$$
$$x(a - y) = a + 1 - y$$
$$x = \frac{a + 1 - y}{a - y}$$

4.

$$a(3b - 1) = 2b + 2$$
$$3ab - a = 2b + 2$$
$$3ab - 2b = 2 + a$$
$$b(3a - 2) = 2 + a$$
$$b = \frac{2 + a}{3a - 2}$$

5.

$$a = \frac{2 - 7b}{3 + 5b}$$
$$a(3 + 5b) = 2 - 7b$$
$$3a + 5ab = 2 - 7b$$
$$5ab + 7b = 2 - 3a$$
$$b(5a + 7) = 2 - 3a$$
$$b = \frac{2 - 3a}{5a + 7}$$

6.

$$n = \frac{1}{2L}\sqrt{\frac{r}{p}}$$
$$n^2 = \frac{1}{4L^2}\left(\frac{r}{p}\right)$$
$$n^2 = \frac{r}{4L^2 p}$$
$$r = 4n^2L^2p$$

Solutions (Evaluation of formulae)**:**

1. $s = \frac{1}{2}(u + v)t$

$= \frac{1}{2}(2.6 + 3.2)(2.5)$

$= \frac{1}{2}(5.8)(2.5)$

$= 7.25$

2. $V = \pi r^2 h$

$275 = \pi r^2(14)$

$$r^2 = \frac{275}{14\pi}$$

$$r = \sqrt{\frac{275}{14\pi}}$$

$r = 2.50$ (rounded to 2 decimal places)

3. $t = a + (n - 1)d$

$25.6 = 5.6 + (n - 1)5$

$25.6 - 5.6 = 5(n - 1)$

$20 = 5(n - 1)$

$4 = n - 1$

$n = 5$

4. $$f = \frac{vu}{v + u}$$

$$20 = \frac{25v}{v + 25}$$

$20(v + 25) = 25v$

$20v + 500 = 25v$

$500 = 25v - 20v$

$500 = 5v$

$v = 100$

5. $$F = \frac{m(v - u)}{t}$$

$$8 = \frac{20(4 - u)}{5}$$

$40 = 20(4 - u)$

$2 = 4 - u$

$u = 2$